

Tutorial 8

Advanced Graph Theory

13th October 2014

1. If $\delta(G) \geq n - 2$ for a simple n -vertex graph G , then $\kappa(G) = \delta(G)$. Prove that this is best possible for each $n \geq 4$ by constructing a simple n -vertex graph with minimum degree $n-3$ and connectivity less than $n - 3$.
2. Prove that Ford-Fulkerson's CSDR Theorem implies Hall's Theorem.
3. Use Menger's Theorem ($\kappa(x, y) = \lambda(x, y)$ when $xy \notin E(G)$), to prove the Konig-Egervary Theorem.
4. Using Menger's Theorem, show that $\kappa(G) = \kappa'(G)$ when G is 3-regular. [Hint: First use the analogue Theorem developed from Menger's Theorem: *If x and y are distinct vertices of a graph G , then the minimum size of an x, y -disconnecting set of edges equals the maximum number of pairwise edge-disjoint x, y -paths.* Then use Menger's Theorem.]